CELT Technical Note 3

How Well Can the CELT Primary Approximate a Parabola?

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Introduction

The CELT primary mirror shape is set to provide a Ritchey-Chretien focus. This is done by allowing the primary and secondary to form a matched set of hyperbolas. The result is that the cassegrain focus has no comatic error, only astigmatism. This great virtue makes a RC design worthwhile, but the hyperbolic primary cannot make a perfect image at prime focus.

Because the primary is segmented, it is interesting to ask how well the primary can approximate a parabola, by suitable piston and tip-tilt motions of the segments.

Baseline Design

The CELT baseline design assumes:

D = 30m (diameter of primary mirror)
k = 90m (radius of curvature of primary mirror)
e = 15m (back focal distance)
f = 450m (final focal length)

From this it follows that for a RC design, (Faber, 1983)

K_1 = -1.002759 (primary mirror conic constant, K = -1 for a parabola)

Further, for the segmented primary we have

a = 0.5m (the radius of the hexagonal segments)

Comparison of Hyperbola and Parabola in the large

The formula for a conic section is given by

$$z(r) = \frac{r^2}{2k} + \frac{(1+K)r^4}{8k^3} + \frac{(1+K)^2 r^6}{16k^5} + \ldots$$

For the values given above, we have at the edge (r = 15m)

z(15) = 1.250m – 23.95µm + 0.92 nm + …

Thus we see that the overall shape difference of the primary between a parabola and the RC hyperbola is about 24µm. This is small compared to the 1mm range of the segment actuators, thus the segments can be moved by the needed amount without unduly stressing the requirements for the segment actuators.

Comparison of the segment shapes

In addition to getting the overall shape correct, individual segment shapes need to be close approximations to those of a parabolic mirror. Thus we need to compare the segment shape with those of a parabolic primary. It is convenient to use the series expansion of the segment shape defined in the segment local coordinates. This expansion is derived by Nelson and Temple-Raston (1982).

In local coordinates, one finds
$z(r) = \alpha_{20} r^2 + \alpha_{22} r^2 \cos(2\theta) + \alpha_{31} r^3 \cos(\theta) + \alpha_{40} r^4 + \ldots$
where the coefficients are given by

$$\alpha_{20} = \frac{a^2}{k}\left[\frac{2 - K\varepsilon^2}{4(1-K\varepsilon^2)^{3/2}}\right] = \frac{a^2}{2k} + \frac{Ka^2\varepsilon^2}{2k} + \frac{9K^2a^2\varepsilon^4}{16k} + \ldots$$

$$\alpha_{22} = \frac{a^2}{k}\left[\frac{K\varepsilon^2}{4(1-K\varepsilon^2)^{3/2}}\right] = \frac{Ka^2\varepsilon^2}{4k} + \frac{3K^2a^2\varepsilon^4}{8k} + \ldots$$

$$\alpha_{31} = \frac{a^3}{k^2}\left[\frac{K\varepsilon\left[1-(K+1)\varepsilon^2\right]^{1/2}(4 - K\varepsilon^2)}{8(1-K\varepsilon^2)^3}\right] = \frac{Ka^3\varepsilon}{2k^2} + \frac{K\ (9K-2)a^3\varepsilon^3}{8k^2} + \ldots$$

$$\alpha_{40} = \frac{a^4}{k^3}\left[\frac{8(1+K) - 24K\varepsilon^2 + 3K^2\varepsilon^4(1-3K) - K^3\varepsilon^6(2-K)}{64(1-K\varepsilon^2)^{9/2}}\right] = \frac{(1+K)a^4}{8k^3} + \frac{3K(1+3K)a^4\varepsilon^2}{16k^3} + \ldots$$

where $\varepsilon = R/k$ and R is the segment off-axis distance. Note that for K = 0 (a sphere) this reduces to the familiar formula for the expansion of a sphere.

Consider the most off axis segments, taken to have R $=R_{max}=$ 15m. The hyperbola-parabola surface is thus given by

$\Delta\alpha_{20} = 0.106\mu m$ $\quad\quad$ $\Delta\alpha_{22} = 0.053\mu m$
$\Delta\alpha_{31} = 0.004\mu m$
$\Delta\alpha_{40} = 0.000\mu m$
or, writing as Zernike Coefficients

$\Delta C_{20} = 0.053\mu m$
$\Delta C_{22} = 0.053\mu m$

Consider the rms surface error, averaged over the entire primary. It is convenient to use Zernike polynomials for this, due to their orthogonality. Considering only the first 2 terms, we get (averaging the segment variances over all the segments)

$$\sigma^2 = \left(\frac{\Delta C_{20}^2}{3} + \frac{\Delta C_{22}^2}{6}\right)\frac{1}{A}\int\left(\frac{r}{R_{max}}\right)^4 dA = \frac{1}{3}\left(\frac{\Delta C_{20}^2}{3} + \frac{\Delta C_{22}^2}{6}\right)$$

or rms = 22nm. $\left(\sim\frac{a^2}{k^3}\right)$

The geometric optics image blur is also of interest. For circular disks of radius a, it is easy to show that the geometric image blur containing 100% of the light is given by

$$\theta_{100} = 16\frac{C_{20}}{a}$$

$$\theta_{100} = 8\frac{C_{22}}{a}$$

Assuming we can approximately add these blurs in quadrature, and average over the primary as before, we can write

$$\theta_{100}^2 = \frac{1}{3}\left(\frac{256C_{20}^2}{a^2} + \frac{64C_{22}^2}{a^2}\right)$$

or $\theta_{100} = 0.23 \text{arc seconds}$

Prime focus coma

Even if the primary mirror can be made perfectly parabolic, prime focus experiences comatic aberrations that grow linearly with field angle. The length of the comatic image, ATC, is given (Faber, 1983) as

$$ATC = -\frac{3\theta}{16F^2} = -0.0833\theta \quad (F = 1.5)$$

so ATC (the total length of the comatic image) is 1 arcseconds, 12 arcseconds off axis. The resulting rms wavefront error (Faber and Nelson, 1982) is

$$wfe = \frac{ATC \bullet D}{18\sqrt{8}} = 2.86\mu m \text{ at 12 arcseconds off axis}$$

Conclusions

Making the CELT primary approximate a paraboloid only requires a small addition to the actuator range. Further, the segment surfaces are fairly close to the desired ones, so for seeing limited observations, the expected image quality is tolerable. In terms of wavefront errors (perhaps of interest for adaptive optics) the wavefront errors at prime focus are also relatively small. However, field angle dependent coma is large, producing a very small field and requiring correction if used with adaptive optics.

References

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Nelson, Jerry, and Temple-Raston, Mark, Off-axis Expansions of Conic Surfaces. (November 1982) Keck Report 91